LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – NOVEMBER 2011

ST 3503/ST 3501/ST 3500 - STATISTICAL MATHEMATICS - II

Date : 01-11-2011 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

PART A (10 X 2 =20)

ANSWER ALL QUESTIONS. EACH CARRIES TWO MARKS.

1. Give an example of a function which is not Rieman integrable.
2. Define the upper integral of a function.
3. What is an improper integral? Give an example.
4. State the Comparison Test for Improper Integrals of II kind.
5. State how the mean and variance are found from the moment generating function (m.g.f).
6. State the recurrence formula for Gamma integral.
7. Give an example of a second order differential equation.
8. Find $L[sin^2 2t]$
9. When do you say a system of linear equations is homogenous? Give an example one such system.
10. If λ is a characteristic root of A, show that λ^2 is a characteristic root of A^2.

PART B (5 X 8 =40)

ANSWER ANY FIVE QUESTIONS. EACH CARRIES EIGHT MARKS.

11. If $f, g \in$ R[a, b] and g is bounded away from zero, show that $\frac{f}{g} \in$ R[a, b].

12. Compute the variance of the random variable X having probability density function (p.d.f)

$$f(x) = \begin{cases} e^{-x}, x > 0 \\ 0, otherwise \end{cases}$$

13. If L[f(t)] = F(s), show that L[t f(t)] = $-\frac{d\,F(s)}{d\,s}$. Hence find L [$t^2 e^{-3t}$] .

14. Show that the improper integral $\int_0^1 \frac{dx}{\sqrt{x(1-x)}}$ converges.

15. If (X, Y) have joint p.d.f

$$f(x,y) = \begin{cases} 2\text{-x-y if } 0 < x < 1, 0 < y < 1 \\ 0 \quad \text{otherwise} \end{cases}$$

find the marginal p.d.f.'s. Also, find Cov(X, Y).

16. Discuss the convergence of Gamma Integral.

17. Solve the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 3y = e^x$

18. State and prove Cayley-Hamilton theorem

PART C

ANSWER ANY TWO QUESTIONS. EACH CARRIES TWENTY MARKS (2 X 20 =40)

19. (a) Show that a function *f* Is Riemann-integrable on [a,b] if and only if for every $\varepsilon > 0$, there is a partition P of [a, b] such that $U(f, P) - L(f, P) < \varepsilon$

(b) State and Prove the First Fundamental Theorem of Integral Calculus. (10 + 10)

20 (a) Discuss the convergence of the following improper integrals:

(i) $\int_0^\infty \frac{1}{x^2+\sqrt{x}}\,dx$ (ii) $\int_{-\infty}^\infty \frac{1}{1+(x-c)^2}\,dx$

(b) Establish the relation between Beta and Gamma integrals. (8 + 12)

21. (a) Solve by using Laplace transforms, the differential equation $\frac{d^2y}{dt^2}+y=t,\ y(0)=1,\ y'(0)=-2$

(b) If the joint p.d.f. of (X_1, X_2) is

$$f(x_1,x_2)=\begin{cases}21\,x_1^2\,x_2^3\ , & 0<x_1<x_2<1\\ 0 \quad , & otherwise\end{cases}$$

find E[$X_1 \mid X_2 = x_2$] and V[$X_1 \mid X_2 = x_2$]. (10 + 10)

22. (a) Solve the following system of equations by matrix inversion method:

$$x-y+z=1$$
$$x+y+z=3$$
$$x+2y-3z=0$$

(b) Find a non-trivial solution for the following system of homogeneous equations:

$$x+2y+5z=0$$
$$x+y+z=0$$
$$3x+5y+11z=0$$

(10 + 10)

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